

# Fibre-Reinforced and Hybrid-Reinforced Concrete: An Updated Bridged Crack Model with Softening Pull-Out Forces

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## Extended Abstract

The Bridged Crack Model is a fracture mechanics approach able to describe the crack propagation process in the critical cross-section of brittle-matrix reinforced members. The model was originally proposed to interpret the fracturing behaviour of steel-bar lightly reinforced concrete (RC) beams [1,2], and it has been recently updated to the case of fibre-reinforced concrete (FRC) [3] by introducing a softening pull-out constitutive law for the reinforcing fibres. Now, a further extension of the model to the case of hybrid-reinforced concrete (HRC) beams—in which the reinforcing phase consists in a combination of continuous steel rebars and short discontinuous fibres—is discussed.

The Bridged Crack Model assumes the concrete matrix as a linear-elastic perfectly-brittle primary phase, its toughening contribution being defined by the fracture toughness,  $K_{IC}$ . On the other hand, nonlinear constitutive laws are assumed to describe the toughening action of the reinforcing secondary phases, which are related to the yielding of steel rebars and to the pull-out of the short fibres. Under these assumptions, it is possible to evaluate the stress-block diagram for each crack depth and to describe the mechanical response in terms of fracturing moment vs localized rotation of the notched cross-section.

Different post-cracking regimes can be predicted by the model, as a function of three scale-dependent dimensionless numbers: the *bar-reinforcement brittleness number*,  $N_P$ , which is directly related to the steel-bar reinforcement percentage,  $\rho$ ; the *fibre-reinforcement brittleness number*,  $N_{P,f}$ , which is directly related to the fibre volume fraction,  $V_f$ ; and the *pull-out brittleness number*,  $N_w$ , which depends on the critical embedment length of the fibre-reinforcement,  $w_c$ . A parametric analysis makes evident how these three dimensionless numbers allow to fully capture the different transitions in the post-cracking regime, which can range from softening to hardening, including hyper-strength phenomena.

The focus of the present work is on the minimum reinforcement condition, i.e., the combination of  $\rho_{min}$  and  $V_{f,min}$  required to guarantee a stable post-peak response, which is defined by the critical values of the two reinforcement brittleness numbers,  $N_P$  and  $N_{P,f}$ . It is found that, at the critical conditions,  $N_P$  and  $N_{P,f}$  can be put in connection with a linear relationship, thus providing an effective tool to the minimum reinforcement design of HRC members, which is still lacking in the current structural design codes [4,5]. The validity of the proposed approach is discussed on the basis of several numerical simulations and supported by experimental campaigns reported in the current scientific literature.

## References

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